Prep. Year | **Mid-Term Exam** | **November 2014**

Total Mark: 20 | **Mathematics 1-A** | **Time: 30 Minutes**

[1]Prove that: $\sinh^{-1} x = \ln(x + \sqrt{1 + x^2})$ and show that it is odd function.

[2]Find (a) $\lim\limits_{x\to 0} \frac{\tan 2x}{1 - 2^{3x}}$ (b) $\lim\limits_{x\to 2} \frac{\ln(x-1)}{2-x}$ (c) $\lim\limits_{x\to\infty} \frac{x + 2^x}{x^2 - 3^{2x}}$

[3] Find **y`** where

(a) $y = 2x^3 - 3\sin 2x$ (b) $y = \tan^{-1}x^2 + \tan^{-2}x$

(c) $y = 5^{x^3} - \tanh^{-1} x$ (d) $y = \cosh x^2 - \operatorname{sech} x$

(e) $y = \ln\cos x + (\ln x)^x$ (f) $y = \sin^{-1} 3^x . \sinh^{-1} x^3$

(g) $y = \sin^5 x. \sin x^5$ (h) $y = \ln \frac{\sqrt[4]{x+\cosh x}.(x+\ln x)^4}{\sqrt[5]{x^2+\cos 2x}.(x-\tan x)^5}$

Model Answer

[1]If $y=\sinh^{-1}x$. Then $\mathrm{x}=\sinh \mathrm{y}=\frac{1}{2}(\mathrm{e^y}-\mathrm{e^{-y}})$. Then $2\mathrm{x}=\mathrm{e^y}-\mathrm{e^{-y}}$.

Then $\mathrm{e^{2y}}-2\mathrm{xe^y}-1=0$. Then $\mathrm{e^y}=\frac{1}{2}[2\mathrm{x}\pm\sqrt{(2\mathrm{x})^2-4(1)(-1)}]=\frac{1}{2}[2\mathrm{x}\pm\sqrt{4\mathrm{x}^2+4}]$

Then $\mathrm{e^y}=\mathrm{x}\pm\sqrt{\mathrm{x}^2+1}$. Then $\mathrm{y}=\ln[\mathrm{x}+\sqrt{\mathrm{x}^2+1}]$

If $f(x)=\sinh^{-1}x=\ln[\mathrm{x}+\sqrt{\mathrm{x}^2+1}]$.

Then $f(-x)=\ln\left[-\mathrm{x}+\sqrt{(-\mathrm{x})^2+1}\right]=\ln\left[-\mathrm{x}+\sqrt{\mathrm{x}^2+1}.\frac{-\mathrm{x}-\sqrt{\mathrm{x}^2+1}}{-\mathrm{x}-\sqrt{\mathrm{x}^2+1}}\right]=\ln\left[\frac{\mathrm{x}^2-(\mathrm{x}^2+1)}{-\mathrm{x}-\sqrt{\mathrm{x}^2+1}}\right]$

Then $f(-x)=\ln\left[\frac{-1}{-\mathrm{x}-\sqrt{\mathrm{x}^2+1}}\right]=\ln\left[\frac{1}{\mathrm{x}+\sqrt{\mathrm{x}^2+1}}\right]=\ln\left[\mathrm{x}+\sqrt{\mathrm{x}^2+1}\right]^{-1}=-\ln[\mathrm{x}+\sqrt{\mathrm{x}^2+1}]$

Then it is odd function.

-- 3.5 Marks

[2](a) $\mathrm{Lim}_{\mathrm{x}\to 0}\ \frac{\tan 2\mathrm{x}}{1-2^{3\mathrm{x}}}=\frac{0}{0}=\mathrm{Lim}_{\mathrm{x}\to 0}-\frac{\tan 2\mathrm{x}}{8^{\mathrm{x}}-1}=-\frac{2}{\ln 8}$

(b) $\mathrm{Lim}_{\mathrm{x}\to 2}\ \frac{\ln(\mathrm{x}-1)}{2-\mathrm{x}}=\frac{0}{0}=\mathrm{Lim}_{\mathrm{x}\to 2}-\frac{\ln(1+(\mathrm{x}-2))}{\mathrm{x}-2}=-1$

(c) $\mathrm{Lim}_{\mathrm{x}\to\infty}\ \frac{\mathrm{x}+2^{\mathrm{x}}}{\mathrm{x}^2-3^{2\mathrm{x}}}=\frac{\infty}{\infty}=\mathrm{Lim}_{\mathrm{x}\to\infty}\frac{\left(\frac{\mathrm{x}}{9^{\mathrm{x}}}\right)-(\frac{2}{9})^{\mathrm{x}}}{\frac{\mathrm{x}^2}{9^{\mathrm{x}}}-1}=\frac{0}{0-1}=0$

-- 4.5 Marks

[3](a) $y=2\mathrm{x}^3-3\sin 2\mathrm{x}$ Then $y`=6\mathrm{x}^2-6\cos 2\mathrm{x}$

(b) $y=\tan^{-1}x^2+\tan^{-2}x$ Then $y`=\frac{2\mathrm{x}}{1+\mathrm{x}^4}-2(\tan \mathrm{x})^{-3}.\sec^2\mathrm{x}$

(c) $y=5^{x^3}+\tanh^{-1}x$ Then $y`=5^{x^3}.\ln 5\,.\,3x^2+\frac{1}{1-\mathrm{x}^2}$

(d) $y=\cosh x^2-\mathrm{sech}\,x$ Then $y`=\sinh x^2\,.\,2x+\mathrm{sech}\,\mathrm{x}\,.\tanh\mathrm{x}$

(e) $y=\ln\cos x+(\ln x)^x$

$=\ln\cos\mathrm{x}+\mathrm{e}^{\mathrm{x}\ln(\ln \mathrm{x})}$ Then $y`=\frac{-\sin x}{\cos x}+\mathrm{e}^{\mathrm{x}.\ln(\ln\mathrm{x})}[\ln(\ln\mathrm{x})+\mathrm{x}\frac{\frac{1}{\mathrm{x}}}{\ln \mathrm{x}}]$

(f) $y=\sin^{-1}3^x\,.\sinh^{-1}x^3$ Then $y`=\sin^{-1}3^x\,.\frac{3x^2}{\sqrt{1+x^6}}+\frac{3^x.\ln 3}{\sqrt{1-3^{2x}}}.\sinh^{-1}x^3$

(g) $\mathrm{y}=\sin^5\mathrm{x}.\sin\mathrm{x}^5$ Then $\mathrm{y}`=\sin^5\mathrm{x}.\cos\mathrm{x}^5\,.\,5\mathrm{x}^4+5\sin^4\mathrm{x}.\cos\mathrm{x}\,.\sin\mathrm{x}^5$

(h) $\mathrm{y}=\ln\frac{\sqrt[4]{\mathrm{x}+\cosh\,\mathrm{x}}.(\mathrm{x}+\ln\mathrm{x})^4}{\sqrt[5]{\mathrm{x}^2+\cos\,2\mathrm{x}}.(\mathrm{x}-\tan\,\mathrm{x})^5}$

$=\frac{1}{4}\ln(\mathrm{x}+\cosh\mathrm{x})+4\ln(\mathrm{x}+\ln\mathrm{x})$

$-\frac{1}{5}\ln(\mathrm{x}^2+\cos 2\mathrm{x})-5\ln(\mathrm{x}-\tan\mathrm{x})$

Then $y`=\frac{1}{4}\frac{1+\sinh x}{x+\cosh x}+4\frac{1+\frac{1}{x}}{x+\ln x}$

$-\frac{1}{5}\frac{2\mathrm{x}-2\sin 2\mathrm{x}}{\mathrm{x}^2+\cos 2\mathrm{x}}-5\frac{1-\sec^2\mathrm{x}}{\mathrm{x}-\tan\mathrm{x}}$

-- 12 Marks

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